RESONANCE PHENOMENON IN THE DIFFRACTION OF A HYDROACOUSTIC WAVE BY A SYSTEM OF CRACKS IN AN ELASTIC PLATE

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In the propagation of acoustic waves in a half space which is filled with a fluid and is covered by an elastic layer, diffraction phenomena arise at inhomogeneities of the layer. These phenomena may be amplified in a resonant manner or attenuated depending on the relative locations of the objects causing the diffraction.

In this paper resonance is examined for the simplest system of the type described: the fluid is covered by a homogeneous elastic plate which is divided into three parts by two straight, parallel cracks of infinitesimal thickness. The incident disturbance is given in the form of a plane monochromatic wave.

In Section 1 a "general" solution (according to the terminology of [1]) is given for the problem of diffraction for any number of defects of arbitrary nature in the plate which are located on parallel straight lines. The diffracted field is found in Section 2 for the case of two cracks and asymptotic simplifications are carried out for low frequencies and large separation of the cracks. In Section 3 the resonant character of the diffraction phenomena is established for a system of this type.

Notation

- U — acoustic potential in the fluid
- W — diffracted part of the acoustic potential
- σ — Poisson's ratio for the plate material
- ρ — fluid density
- ρ_0 — density of the plate material
- c — wave velocity in the fluid
- c_t — velocity of transverse waves in plate material
- H — plate thickness
- k — dimensionless wave number in the fluid ($k = 2\pi H/\Lambda$)
- Λ — wave length in the fluid
- φ_0 — angle at which the incident wave moves (measured from the positive direction of the Ox-axis)

1. An ideal compressible fluid fills a half space which is covered by an elastic plate. There are certain number of parallel rectilinear slits in the plate. A plane monochromatic wave is incident from the depths of the fluid; the direction of motion of the wave is orthogonal to the direction of the slits. It is required to find the field created by the wave.

With a proper choice of coordinate axes (Fig.1) this problem is a two-dimensional one. Its mathematical formulation is as follows [1 and 2].

It is required to find the solution of the Helmholtz equation

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + k^2 U = 0 \qquad (-\infty < x < +\infty,\ 0 < y < +\infty) \tag{1.1}$$

which is continuous throughout up to the x-axis. On the x-axis it is assumed that the following boundary condition is satisfied:

$$LU \equiv \frac{\partial^5 U(x, 0)}{\partial x^4 \partial y} - \delta_0 k^2 \frac{\partial U(x, 0)}{\partial y} + \nu_0 k^2 U(x, 0) = \sum_{n=1}^{m} \sum_{s=1}^{4} A_{sn} \delta^{(s-1)}(x - a_n) \tag{1.2}$$

Here

$$\delta_0 = 6(1 - \sigma) \frac{c^2}{c_t^2}, \qquad \nu_0 = 6(1 - \sigma) \frac{\rho c^2}{\rho_0 c_t^2}$$

where m is the number of cracks, a_n is the x-coordinate of the nth crack, and $\delta(x)$ is the Dirac delta function. The constants A_{sn} are such that the boundary-contact relations

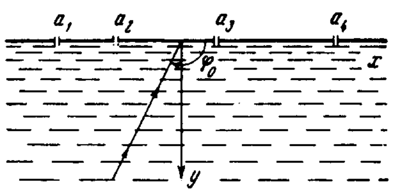

Fig. 1

$$\lim \frac{\partial^3 U(x, 0)}{\partial x^2 \partial y} = 0, \qquad \lim \frac{\partial^4 U(x, 0)}{\partial x^3 \partial y} = 0$$

$$\text{for} \quad x \to a_n \pm 0 \quad (n = 1, \ldots, m) \tag{1.3}$$

hold.

Moreover, the difference $U - U_0$, where

$$U_0 = \exp(i\varkappa x - i\sqrt{k^2 - \varkappa^2}\, y) \qquad (\varkappa = -k \cos \varphi_0) \tag{1.4}$$

must satisfy the principle of limiting absorption. The quantities x and y are considered to be dimensionless. The transformation to x and y from the corresponding dimensional quantities is accomplished by dividing by the thickness H on the plate.

The boundary condition (1.2) requires explanation inasmuch as the expression used for its right side has not been employed before. In the formulation of the corresponding problem [2] for a single crack located at $x = 0$ the boundary conditions (using the notation of the present paper) had the form

$$LU = 0 \qquad (x \neq 0)$$

The wave field which was found as a result of solving the problem was written in the following manner:

$$U = U_0 + U_1 + W, \qquad L(U_0 + U_1) = 0$$

$$W = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{a\lambda^3 + b\lambda^2 + c\lambda + d}{(\lambda^4 - \delta_0 k^2)\sqrt{k^2 - \lambda^2} - i\nu_0 k^2} \exp\left(i\lambda x + i\sqrt{k^2 - \lambda^2}\, y\right) d\lambda$$

Applying the operator L and U we arrive at Equation

$$LU = LW = ib\delta'''(x) - b\delta''(x) - ic\delta'(x) + d\delta(x)$$

which coincides with (1.2) for $m = 1$ and $a_1 = 0$ except for the notation of the constants.

It is natural to consider that the character of the singularity of LU at a crack is not altered by the presence of other defects in the plate which are isolated from the crack.

By direct verification it is easy to demostrate that the solution of the problem which has been posed has the following form

$$U = U_0 + U_1 + W \tag{1.5}$$

where

$$U_1 = \frac{l^*(\varkappa)}{l(\varkappa)} \exp\left(i\varkappa x + i\sqrt{k^2 - \varkappa^2}\, y\right) \qquad \left(l(\lambda) = (\lambda^4 - \delta_0 k^2)\sqrt{k^2 - \lambda^2} - i\nu_0 k^2\right) \tag{1.6}$$

$$W = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{1}{l(\lambda)} \sum_{n=1}^{m} e^{-ia_n\lambda} \sum_{s=1}^{4} A_{sn} (i\lambda)^{s-1} \exp\left(i\lambda x + i\sqrt{k^2 - \lambda^2}\, y\right) d\lambda \tag{1.7}$$

Here U_1 represents the wave which is reflected from the plate, W is the diffracted field which is caused by the presence of the cracks in the plate, l^* is the complex conjugate of l . The choice of the branch of $l(\lambda)$ is clear from Fig. 2, where the solid line denotes the contour of integration and the dashed lines are cuts in the complex plane λ . The radical $\sqrt{k^2 - \lambda^2}$ is taken as positive on the segment of the real axis $(-k, k)$. The numbers $\pm\lambda_s$ (s = 0, 1, 2, 3, 4) are the roots of the function $l(\lambda)$; their distribution is described in [2]. Only those roots of $l(\lambda)$ which lie on the sheet of the Riemann surface which is considered are depicted in Fig. 2.

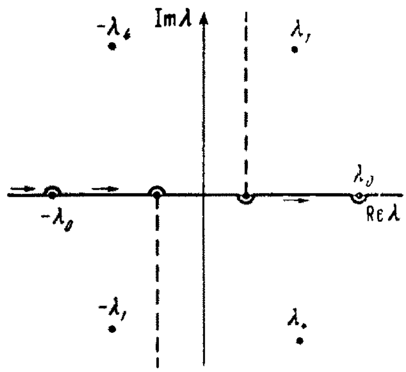

Fig. 2

It should be noted that the general solution given by Formulas (1.5), (1.4), 1.6) and (1.7) is valid for finding the field in the predence of any disturbances in the mechanical properties of the plate at the points $x = a_n$ (e.g. hinged connections, see [3]), and not only for the case of cracks. The physical behavior at $x = a_n$ is taken into account by the boundary-contact conditions. Therefore, before the relations (1.3) are introduced, the solution which has been written out has a universal character in the sense indicated.

Conditions (1.3) generate an inhomogeneous system of $4m$ linear equations for finding the $4m$ unknown constants A_{sn} . On physical grounds it may be assumed that this system has a unique solution for any k . Computations will be given below corresponding to the case of two cracks.

2. We shall denote the distance between the cracks by $2a$ (as before, the transformation to the dimensionless distance is accomplished by dividing by H) and we shall place the origin of coordinates at the center of the segment between the cracks. The expression for the diffracted field has the form

$$W = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{1}{l(\lambda)} \Big[e^{-ia\lambda} \sum_{s=1}^{4} A_{s1} (i\lambda)^{s-1} + + e^{ia\lambda} \sum_{s=1}^{4} A_{s2} (i\lambda)^{s-1} \Big] \exp (i\lambda x + \sqrt{k^2 - \lambda^2} y) \, d\lambda \qquad (2.1)$$

We now split the wave field U and its components U_0, U_1 and W into their symmetric and antisymmetric parts with respect to the variable x

$$U_i = U_i^+ + U_i^-, \qquad U_i^{\pm} (x, y) = {}^1/_2 \, [U_i (x, y) \pm U_i (-x, y)] \qquad (2.2)$$

The problem of finding the diffracted field is thereby divided into two problems, one for each part. Only four constants will occur in each of these problems. With this breakdown the components of the field have the form

$$U_0^{\pm} = \begin{Bmatrix} \cos \varkappa x \\ i \sin \varkappa x \end{Bmatrix} e^{-i\sqrt{k^2 - \varkappa^2} y}, \qquad U_1^{\pm} = \frac{l^* (\varkappa)}{l (\varkappa)} \begin{Bmatrix} \cos \varkappa x \\ i \sin \varkappa x \end{Bmatrix} e^{i\sqrt{k^2 - \varkappa^2} y} \qquad (2.3)$$

$$W^{\pm} = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{b_1^{\pm} + b_2^{\pm}\lambda + b_3^{\pm}\lambda^2 + b_4^{\pm}\lambda^3}{l(\lambda)} \begin{Bmatrix} \cos \lambda x \\ i \sin \lambda x \end{Bmatrix} \exp (i\lambda a + i \sqrt{k^2 - \lambda^2} \, y) \, d\lambda$$

Here and in what follows, the upper line in the expressions in braces should be used for the symmetric part of the field and the lower line for the antisymmetric part. Analogously, in using the double signs the upper one refers to the symmetric part of the field, the lower to the antisymmetric part.

In the process of satisfying the boundary-contact conditions it is necessary to carry out differentiation under the integral sign in the expression for $W\pm$. After this is done and y goes to zero, divergent integrals result, the use of which is justified in [1].

After some simple computations analogous to those described in [2], it is possible to arrive at the system

$$\begin{gathered} (Q_2 \pm P_2) b_1^{\pm} + Q_3 b_2^{\pm} + (Q_4 \pm P_4) b_3^{\pm} + Q_5 \, b_4^{\pm} = \beta \begin{cases} \cos \varkappa a \\ i \sin \varkappa a \end{cases} \\ P_3 b_2^{\pm} + P_5 b_4^{\pm} = 0 \\ Q_3 b_1^{\pm} + (Q_4 \mp P_4) \, b_2^{\pm} + Q_5 b_3^{\pm} + (Q_6 \mp P_6) \, b_4^{\pm} = \varkappa \beta \begin{cases} i \sin \varkappa a \\ \cos \varkappa a \end{cases} \\ P_3 b_1^{\pm} + P_5 b_3^{\pm} = 0 \end{gathered} \qquad (2.4)$$

where

$$P_n = \frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{\lambda^n\sqrt{k^2-\lambda^2}}{l(\lambda)}e^{i0\lambda}\,d\lambda, \qquad Q_n = \frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{\lambda^n\sqrt{k^2-\lambda^2}}{l(\lambda)}e^{i2a\lambda}\,d\lambda$$

$$\beta = \frac{2\nu_0 k^2\varkappa\sqrt{k^2-\varkappa^2}}{l(\varkappa)} \qquad (2.5)$$

Here the following abbreviated notation is used

$$\int_{-\infty}^{\infty} f(\lambda)\,e^{i0\lambda}\,d\lambda = \lim_{x\to+0}\int_{-\infty}^{\infty} f(\lambda)\,e^{i\lambda x}\,d\lambda$$

An asymptotic approximation of the solution of the system (2.4) will be obtained for $k \ll 1$. This inequality serves as the condition for the possibility of treating an elastic layer on the surface of the fluid as a plate. Therefore, the leading term of the asymptotic expansion of W in the small parameter k determines to a considerable extent the behavior of the diffracted field when the model is physically reasonable and for sufficiently small k it practically coincides with the actual diffracted field. The parameter a, in addition to k, also plays an essential role in the representation which will be obtained below. We remark that the presence in the problem of a characteristic linear dimension which is comparable to the wave length (it is just this case for the distance between cracks, which presents the greatest interest) does not permit us to consider the asymptotic solution sought as a long-wave one in the usual sense.

In (2.5) we perform the change of variable $\lambda = \nu_0^{1/5}k^{3/5}\mu$ and obtain

$$P_n = \nu_0^{\frac{n-3}{5}}k^{\frac{2(n-3)}{5}}p_n, \qquad p_n = \frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{\mu^n(\mu^2-\gamma k^{6/5})^{1/2}e^{i0\mu}\,d\mu}{(\mu^4-\alpha k^{2/5})(\mu^2-\gamma k^{6/5})^{1/2}-1}$$

$$(n = 2, 3, 4, 5, 6)$$

$$Q_n = \nu_0^{\frac{n-3}{5}}k^{\frac{2(n-3)}{5}}q_n, \qquad q_n = \frac{1}{4\pi}\int_{-\infty}^{\infty}\frac{\mu^n(\mu^2-\gamma k^{6/5})^{1/2}e^{if\mu}\,d\mu}{(\mu^4-\alpha k^{2/5})(\mu^2-\gamma k^{6/5})^{1/2}-1} \qquad (2.6)$$

where

$$f = 2a\nu_0^{1/5}k^{3/5}, \qquad \alpha = \delta_0\nu_0^{-4/5}, \qquad \gamma = \nu_0^{-3/5}, \qquad \sqrt{\mu^2-\gamma k^{6/5}} > 0 \quad (2.7)$$

$$\text{for } \mu > \gamma^{1/2}k^{3/5}$$

The integrals for p_n can be expressed in terms of elementary functions with the aid of calculations similar to those described in [2]. From these we obtain

$$p_{2n+1} = \frac{i}{4}\sum_{s=0}^{4}\frac{\mu_3^{2n}(\mu_3^2-\gamma k^{6/5})}{5\mu_s^4-4\mu_s^2\gamma k^{6/5}-\alpha k^{2/5}} \qquad (n = 1, 2)$$

$$p_{2n} = -\frac{1}{2\pi}\sum_{s=0}^{4}\frac{\mu_s^{2n-1}(\mu_s^2-\gamma k^{6/5})}{5\mu_s^4-4\mu_s^2\gamma k^{6/5}-\alpha k^{2/5}}\left(\ln\frac{\mu_s+(\mu_s^2-\gamma k^{6/5})^{1/2}}{\gamma^{1/2}k^{3/5}} - i\frac{\pi}{2}\right)$$

$$(n = 1, 2, 3) \qquad (2.8)$$

where μ_s $(s = 0, 1, 2, 3, 4)$ are roots of the equation

$$(\mu_s^4 - \alpha k^{2/5})^2 (\mu_s^2 - \gamma k^{4/5}) - 1 = 0, \qquad \nu_0^{1/5} k^{2/5} \mu_s = \lambda_s$$

The choice of the branch of the logarithm is fixed by the requirements

$$0 \leqslant \operatorname{Im} \ln N < 2\pi, \ \operatorname{Re} \ln N > 0$$

There is a Taylor expansion for μ_s in powers of $k^{2/5}$ in the form

$$\mu_s = e^{\frac{2\pi i s}{5}} + \frac{1}{5} \alpha k^{2/5} e^{\frac{4\pi i s}{5}} - \frac{1}{25} \alpha^2 k^{4/5} e^{\frac{6\pi i s}{5}} + \left(\frac{1}{125} \alpha^3 + \frac{1}{10} \gamma\right) k^{6/5} e^{\frac{8\pi i s}{5}} + \ldots \quad (2.9)$$

On the basis of Formulas (2.8) and (2.9) it may easily be concluded that p_{2n+1} is expandable in a Taylor series in integral powers of k^2 and that p_{2n} is expressible in the form of a specific expansion in which $\ln k$ and powers of $k^{2/5}$ occur.

The actual computations lead to the result

$$p_3 = \frac{i}{4} + O(k^2), \quad p_5 = O(k^4)$$

$$p_{2n} = \frac{i}{5} \{1 - \exp [{}^2/_5\, \pi i\, (2n - 3)]\}^{-1} + O(k^{2/5}) \quad (n = 1, 2, 3) \qquad (2.10)$$

From (2.10) and homogeneous equalities of the system (2.4) it is easy to ascertain that the constants $b_1\pm$, $b_2\pm$ are of high order in k and do not contribute to the first term of the asymptotic representation for W. In this sense the system (2.4) may be reduced and, taking account of the obvious asymptotic simplifications of the right-hand sides, we may rewrite it in the form

$$(Q_4 \pm P_4)\, b_3^\pm + Q_5\, b_4^\pm = 2ik^3 \cos^2 \varphi_0 \sin \varphi_0 \begin{cases} \cos \varkappa a \\ i \sin \varkappa a \end{cases}$$

$$Q_5 b_3^\pm + (Q_6 \mp P_6)\, b_4^\pm = 0 \qquad (2.11)$$

We deform the contour of integration for q_n into a loop which surrounds the upper branch cut and reduce the integral along this entire path to one, on only the right edge of the cut. If account is taken of the residues at the poles μ_0, $\mu_1 - \mu_4$ which are crossed as the contour is deformed it is easy to obtain expression for q_n which does not contain a divergent integral and which is convenient for finding the necessary asymptotic representations.

$$q_n = \frac{i}{2} \sum_{\mu_s = \mu_0,\, \mu_1, - \mu_4} \frac{\mu_s^{n-1} (\mu_s^2 - \gamma k^{4/5})\, e^{i\mu_s f}}{5\mu_s^4 - 4\mu_s^2 \gamma k^{4/5} - \alpha k^{2/5}} + \frac{1}{2\pi} \int\limits_{\gamma^{1/2} k^{2/5}}^{\gamma^{1/2} k^{2/5} + i\infty} \frac{\mu^n (\mu^2 - \gamma k^{4/5})^{1/2} e^{if\mu}\, d\mu}{(\mu^4 - \alpha k^{2/5})^2 (\mu^2 - \gamma k^{4/5}) - 1} \quad (2.12)$$

$$(n = 2, 3, 4, 5, 6)$$

The form of the asymptotic representation for q_n depends strongly on the magnitude of the parameter f.

By virtue of the fact that the relation

$$\lambda_0 = \nu_0^{1/5} k^{2/5} \mu_0 \approx \nu_0^{1/5} k^{2/5}$$

holds for the dimensionless wave number λ_0 of the flexural wave (see (2.9)), the parameter f is approximately equal to 2π times the ratio of the distance between the cracks to the length of the flexural wave. (The exact value of this expression will hereafter be denoted by F).

Taking f (or F, which is equivalent) as a large parameter of the problem, we can deduce Equation

$$q_n = \frac{i}{10} e^{iF} + O\left(k^{2/5}\right) + O\left(\frac{1}{F^{n+2}}\right) \qquad (F = 2a\lambda_0 = f\mu_0) \qquad (2.13)$$

on the basis of (2.9) and (2.12).

If the exact value of μ_0 in this equation is replaced by the first s terms of the series, an additional error $O\left(ak^{2/5(2s+2)}\right)$ results.

When f is not a large number, Formula

$$q_n = \frac{i}{10}\left\{e^{if} + \exp\left[\frac{2\pi i}{5}(n-3) + ife^{2/5\pi i}\right] + \right.$$

$$\left. + \exp\left[\frac{3\pi i}{5}(n-3) + ife^{3/5\pi i}\right]\right\} + \frac{i^n}{2\pi}\int_0^\infty \frac{\tau^{n+1}}{\tau^{10}+1} e^{-f\tau}\, d\tau + O(k^{2/5}) \qquad (2.14)$$

holds.

We now turn to the case when f is large. Solving the system (2.11) and taking account of the asymptotic equalities (2.10) and (2.13), we arrive at the following values for the unknown constants:

$$b_3^{\pm} = \mp 20\, i \sin \frac{\pi}{5}\, \frac{e^{iF}\cos 0.1\pi \mp e^{-0.1i\pi}}{e^{iF}\cos 0.1\pi \mp e^{-0.9i\pi}} \begin{Bmatrix}\cos \varkappa a \\ i \sin \varkappa a\end{Bmatrix} \frac{\cos^2\varphi_0 \sin\varphi_0 k^{11/5}}{\nu_0^{3/5}}$$

$$b_4^{\pm} = \pm 20\, i \sin \frac{\pi}{5}\, \frac{e^{iF}\cos 0.1\pi}{e^{iF}\cos 0.1\pi \mp e^{-0.9i\pi}} \begin{Bmatrix}\cos \varkappa a \\ i \sin \varkappa a\end{Bmatrix} \frac{\cos^2\varphi_0 \sin\varphi_0 k^{11/5}}{\nu_0^{3/5}} \qquad (2.15)$$

Equations (1.5), (2.2) and (2.15) (together with the fact that $b_1^{\pm}, b_2^{\pm} = 0$) constitute the solution of the problem which has been stated, for the asymptotic situation considered. This solution is studied below with the aim of obtaining its physical consequences.

3. The main components of the diffracted field W under study are a cylindrical wave W_0 and two surface waves, one of which (the direct wave W_+) moves in the direction of increasing coordinate x, and the other one (the reverse wave W_-) which moves in the oposite direction. We shall agree to use subscripts in the designation of these waves; the superscripts plus and minus will be retained for denoting the even and odd parts of the field and its components. We also introduce the superscript $^{\circ}$ with which we shall denote elements of the wave field for the problem of diffraction due to a single crack, as solved in [2]. The ratio of two corresponding elements of the wave field for the present problem and the problem with one crack will be called an "influence function" and will be denoted by R .

An influence function shows how the presence of the second crack affects the amplitude and phase of the corresponding wave phenomenon.

In what follows, the expression for the diffracted field will be taken as

$$W = W^{+} + W^{-} \tag{3.1}$$

$$W^{\pm} = \frac{1}{4\pi i}\int_{-\infty}^{\infty} \frac{e^{i\lambda a}(b_3^{\pm}\lambda^2 + b_4^{\pm}\lambda^3) \pm e^{-i\lambda a}(b_3^{\pm}\lambda^2 - b_4^{\pm}\lambda^3)}{l(\lambda)} \exp(i\lambda x + i\sqrt{k^2-\lambda^2}y)\,d\lambda$$

The direct surface wave W_+ is extracted from (3.1) by taking the residue of the integrand at $\lambda = \lambda_0$. After some calculations we have

$$W_+ = -2e^{i(0.1\pi-0.5F)}\sin\frac{\pi}{5}\left\{\frac{e^{iF} - e^{-0.2i\pi}}{e^{iF}\cos 0.1\pi - e^{-0.3i\pi}}\cos \varkappa a + \right.$$

$$\left. + \frac{e^{iF} + e^{-0.2i\pi}}{e^{iF}\cos 0.1\pi + e^{-0.3i\pi}}\, i\sin \varkappa a\right\}\frac{\cos^2\varphi_0 \sin\varphi_0 k^{9/4}}{\nu_0^{3/4}}\, e^{i\lambda_0 x - \lambda_0 y} \tag{3.2}$$

The expression for the direct surface wave W_+° in the case of a single crack can be written in the form [2]

$$W_+^{\circ} = -2e^{-0.2i\pi}\sin\frac{\pi}{5}\,\frac{\cos^2\varphi_0\sin\varphi_0 k^{9/4}}{\nu_0^{3/4}}\, e^{i\lambda_0 x-\lambda_0 y} \tag{3.3}$$

Comparing Equations (3.2) and (3.3) we arrive at the following expression for the influence function for the direct surface wave:

$$R_+ = e^{-i(0.5F+0.1\pi)}\left\{\frac{e^{iF} - e^{-0.2i\pi}}{e^{iF}\cos 0.1\pi - e^{-0.3i\pi}}\cos(ka\cos\varphi_0) - \right.$$

$$\left. - \frac{e^{iF} + e^{-0.2i\pi}}{e^{iF}\cos 0.1\pi + e^{-0.3i\pi}} - i\sin(ka\cos\varphi_0)\right\} \tag{3.4}$$

As is apparent from (3.4), the function R_+ depends periodically on two arguments: the distance between cracks measured in wave lengths of the flexural wave (the parameter F) and the phase difference of the incident wave at the left and right cracks (the parameter $ka\cos\varphi_0$). As a result of this, the absolute value of R_+ undergoes considerable oscillation.

As an illustration let us examine the case when the phase shift of the incident wave between the cracks is negligibly small or else is a multiple of 2π

$$ka\cos\varphi_0 = 2s\pi$$

Then

$$R_+ = e^{-i(0.5F+0.1\pi)}\frac{e^{iF} - e^{-0.2i\pi}}{e^{iF}\cos 0.1\pi - e^{-0.3i\pi}} \tag{3.5}$$

and R_+ goes to zero for $F = -0.2\pi + 2n\pi$.

Let us denote the dimensionless length of the flexural wave by l_0 . In the approximation under study it has been found that for

$$\frac{2a}{l_0} = n - \frac{1}{10} \qquad (ka\cos\varphi_0 = 2s\pi,\ n,\ s \text{ are integers}) \qquad (3.6)$$

the direct surface wave disapears completely. It is curious that quenching of the direct surface wave is obtained for a nonintegral number of flexural waves $(n - 0.1)$ distributed in the interval between cracks.

The modulus of the influence function R_+ attains its maximum near points where the modulus of the denominator is smallest. This maximum is approximately equal to seven. In other words, the presence of the second crack when

$$\frac{2a}{l_0} \approx n - \frac{3}{20} \qquad (ka\cos\varphi_0 = 2s\pi;\ n,\ s \text{ are integers}) \qquad (3.7)$$

causes an approximately sevenfold amplification in the amplitude of the direct surface wave.

Thus, in this system a very strong resonance phenomenon is present for the surface wave.

Let us now turn to the cylindrical wave W_0 . This wave is extracted from (3.1) with the aid of the method of stationary phase

$$W_0 = V(\varphi)\frac{e^{ikr}}{\sqrt{kr}} \qquad (3.8)$$

$$V(\varphi) = \frac{20}{\sqrt{2\pi}} e^{5/4\, i\pi} \sin\frac{\pi}{5}\left\{\frac{e^{iF}\cos 0.1\pi - e^{-0.1i\pi}}{e^{iF}\cos 0.1\pi - e^{-0.3i\pi}}\cos(ka\cos\varphi_0)\cos(ka\cos\varphi) - \right.$$

$$\left. - \frac{e^{iF}\cos 0.1\pi + e^{-0.1i\pi}}{e^{iF}\cos 0.1\pi + e^{-0.3i\pi}}\sin(ka\cos\varphi_0)\sin(ka\cos\varphi)\right\}\frac{\cos^2\varphi_0\sin\varphi_0\cos^2\varphi\sin\varphi\, k^{13/5}}{\nu_0^{6/5}}$$

$$(x = r\cos\varphi,\ y = r\sin\varphi)$$

In the case of a single crack the cylindrical wave is determined by Formula

$$W_0^\circ = V^\circ(\varphi)\frac{e^{ikr}}{\sqrt{kr}} \qquad (3.9)$$

$$V^\circ(\varphi) = \frac{10}{\sqrt{2\pi}}\exp\left(i\,\frac{29}{20}\pi\right)\sin\frac{\pi}{5}\;\frac{\cos^2\varphi_0\sin\varphi_0\cos^2\varphi\sin\varphi\, k^{13/5}}{\nu_0^{6/5}}$$

We obtain the following expression for the influence function:

$$R_0(\varphi) = 2e^{-0.2i\pi}\left\{\frac{e^{iF}\cos 0.1\pi - e^{-0.1i\pi}}{e^{iF}\cos 0.1\pi - e^{-0.3i\pi}}\cos(ka\cos\varphi_0)\cos(ka\cos\varphi) - \right.$$

$$\left. - \frac{e^{iF}\cos 0.1\pi + e^{-0.1i\pi}}{e^{iF}\cos 0.1\pi + e^{-0.3i\pi}}\sin(ka\cos\varphi_0)\sin(ka\cos\varphi)\right\} \qquad (3.10)$$

Since the angle φ is among the arguments of the influence function, the character of the directional pattern of the cylindrical wave holding in the case of one crack can be distorted considerably for the cracks. In accordance with the values of the parameters F and $ka \cos \varphi_0$ not only will resonant amplification or attenuation of the intensity of the cylindrical radiation take place, but also the two main lobes of the directional pattern will be split up.

As an example let us consider the case when $ka \cos \varphi_0 = n\pi$. Then

$$R_0 = 2(-1)^n e^{-0.2i\pi}\frac{e^{iF}\cos 0.1\pi - e^{-0.1i\pi}}{e^{iF}\cos 0.1\pi - e^{-0.3i\pi}}\cos(ka\cos\varphi) \qquad (3.11)$$

It is clear that in this case each of the two main lobes of the directional pattern splits up into $E(n/\cos \varphi_0) + 1$ smaller lobes ($E(x)$ denotes the integral part of x). It is interesting to note that the cylindrical radiation does not dissapear for any F . The ratio of the amplitude of the maximum value of the cylindrical wave to the minimum, amounts in this case to a quantity of the order of 140.

Let us recall that the conclusions of this section hold in the asymptotic sense for large values of the parameter F . In case resonant phenomena in the system which has been described must be investigated for values of F which cannot be regarded as large (for example, to find the first maximum of the modulus of an influence function), it is necessary to resort to numerical computations based on Equation (2.14).

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